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Entropy of a Uniformly Accelerating Black Hole

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Kinnersley has discussed the space–time of an arbitrarily accelerating point mass. We select a simple case in which the black hole is uniformly accelerated and the mass does not vary with time. We adopt thin film brick-wall model to calculate the entropy of black hole. We find that both the temperature and the entropy density of black hole can be calculated at every point on the horizon. This result indicates that the conclusion that black hole entropy is proportional to its area can be applied to horizon not only globally, but also locally.

KEY WORDS: entropy; accelerating black hole; brick-wall model.

1. INTRODUCTION

Since Bekenstein suggested that the entropy of a black hole is proportional to its surface area, the concerned research work has got much progress (Bekenstein, 1974; Gibbons and Hawking, 1977; Hawking, 1975). The brick-wall model suggested by ’t Hooft (1985) gives a statistical explanation to the origin of black hole entropy. Recently, the brick-wall model is developed to the thin film brick-wall model (Liu and Zhao, 2001). In this paper, we adopt the thin film brick-wall model to calculate the entropy of a uniformly accelerating black hole.

Kinnersley (1969) has discussed the space–time of an arbitrarily accelerating point mass. The metric used in this paper is a simple case in which the black hole is uniformly accelerated and the mass does not vary with time. In Section 2 we give the line element of the space–time and the surface equation of the horizon. Because the black hole is accelerated, the horizon is axisymmetric. Unlike the spherically symmetric black hole, the different points on the horizon may have different temperature. In Section 3 we use a method which is proposed by Zhao and Dai (1992) to study the temperature of the black hole. It is obvious that in this space–time the thermal equilibrium does not exist in large region, and the standard

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brick-wall method encounters difficulty in this situation. The thin film brick-wall model is adopted to overcome this difficulty. We explain the thin film brick-wall model in Section 4 and calculate the entropy of the Rindler horizon as an example. The entropy of the uniformly accelerating black hole is studied in Section 5 in detail, and we give conclusion and discussion in Section 6.

2. METRIC OF UNIFORMLY ACCELERATING BLACK HOLE

The metric of an arbitrarily accelerating point mass has been derived by Kinnersley (1969):

$$\begin{aligned} ds^2 &= [1 - 2ar\cos\theta - r^2(f^2 + h^2\sin^2\theta) - 2Mr^{-1}]\,du^2 + 2du\,dr \\ &\quad + 2r^2 f\,du\,d\theta + 2r^2 h\sin^2\theta\,du\,d\varphi - r^2 d\theta^2 - r^2\sin^2\theta\,d\varphi^2, \end{aligned} \tag{1}$$

where

$$\begin{aligned} f &= -a(u)\sin\theta + b(u)\sin\varphi + c(u)\cos\varphi, \\ h &= b(u)\cot\theta\cos\varphi - c(u)\cot\theta\sin\varphi. \end{aligned} \tag{2}$$

a, b, c, and M are the functions of the retarted Eddington coordinate u. a is the value of acceleration. b and c describe the variation ratio of the acceleration's direction. In the case of uniform acceleration, $a =$ const., and $b = c = 0$. Furthermore, if M does not vary with time, the metric can be reduced to

$$\begin{aligned} ds^2 &= (1 - 2ar\cos\theta - r^2 f^2 - 2Mr^{-1})\,du^2 + 2\,du\,dr + 2r^2 f\,du\,d\theta \\ &\quad - r^2\,d\theta^2 - r^2\,\sin^2\theta\,d\varphi^2, \end{aligned} \tag{3}$$

where

$$f = -a\,\sin\theta. \tag{4}$$

Replace the retarded Eddington coordinate u with the advanced Eddington coordinate v, and adopt $(-, +, +, +)$ signature; then the metric becomes

$$\begin{aligned} ds^2 &= -(1 - 2ar\cos\theta - r^2a^2\sin\theta - 2Mr^{-1})\,dv^2 + 2dv\,dr \\ &\quad - 2r^2a\sin\theta\,dv\,d\theta + r^2\,d\theta^2 + r^2\sin^2\theta\,d\varphi^2. \end{aligned} \tag{5}$$

$\theta = 0$ is the direction of acceleration and the space–time is axially symmetrical. The determinant of the metric is

$$g = -r^4\sin^2\theta, \tag{6}$$

and the nonzero contravariant components of the metric are

$$\begin{aligned} g^{01} &= g^{10} = 1, \\ g^{11} &= 1 - 2ar\cos\theta - 2Mr^{-1}, \\ g^{12} &= g^{21} = a\sin\theta, \\ g^{22} &= r^{-2}, \\ g^{33} &= r^{-2}\sin^{-2}\theta. \end{aligned} \tag{7}$$

Now let us find the horizon equation of the space–time represented by Eq. (5). Considering uniform acceleration, $M =$ const., and axial symmetry, the surface equation of the event horizon can be written as

$$H = H(r,\theta) = 0 \quad \text{or} \quad r = r(\theta), \tag{8}$$

which should satisfy null surface qualification

$$g^{\mu\nu}\frac{\partial H}{\partial x^{\mu}}\frac{\partial H}{\partial x^{\nu}} = 0. \tag{9}$$

Substituting Eq. (7) into Eq. (9), we get

$$\left(1 - 2ar\cos\theta - \frac{2M}{r}\right)\left(\frac{\partial H}{\partial r}\right)^2 + 2a\sin\theta\frac{\partial H}{\partial r}\frac{\partial H}{\partial\theta} + \frac{1}{r^2}\left(\frac{\partial H}{\partial\theta}\right)^2 = 0. \tag{10}$$

From Eq. (8), we have

$$\frac{\partial H}{\partial r}\frac{\partial r}{\partial\theta} + \frac{\partial H}{\partial\theta} = 0. \tag{11}$$

Substituting Eq. (11) into Eq. (10), we get

$$1 - 2ar_H\cos\theta - \frac{2M}{r_H} - (2a\sin\theta)r'_H + \frac{r'^2_H}{r^2_H} = 0, \tag{12}$$

where

$$r'_H = \left(\frac{\partial r}{\partial\theta}\right)_{r=r_H}. \tag{13}$$

Surface r_H that satisfies Eq. (12) is the event horizon of the uniformly accelerating black hole.

If we let M in Eq. (5) be zero, the space–time is simplified to Rindler space–time relative to a uniformly accelerating observer. The metric is reduced to

$$\begin{aligned} ds^2 = {} & -(1 - 2ar\cos\theta - r^2a^2\sin^2\theta)dv^2 + 2dv\,dr \\ & - 2r^2a\sin\theta\,dv\,d\theta + r^2\,d\theta^2 + r^2\sin^2\theta\,d\varphi^2. \end{aligned} \tag{14}$$

The horizon equation is reduced to

$$1 - 2ar_H \cos\theta - (2a\sin\theta)r'_H + \frac{r'^2_H}{r^2_H} = 0. \tag{15}$$

The solution of Eq. (15) is a paraboloid of revolution

$$r_H = \frac{1}{a(1+\cos\theta)}. \tag{16}$$

3. TEMPERATURE OF BLACK HOLE

In this section, we use a method which is proposed by Zhao and Dai (1992) to study the temperature of the black hole. The calculation of this method is simple and precise. It is fit for various black holes, including nonspherically symmetric black holes and nonasymptotically flat black holes.

This new method is based on the Damour–Ruffini's scheme (Damour and Ruffini, 1976). The essential point is that if we use the tortoise coordinate, the radial part of the Klein–Gordon equation near the horizon has the standard form of the wave equation

$$\frac{\partial^2\Phi}{\partial r_*^2} + 2\frac{\partial^2\Phi}{\partial v\,\partial r_*} = 0. \tag{17}$$

This means that in the case of accelerating black hole the two-dimension line element of space–time is obviously conformal with Minkowski space–time. We can introduce parameter κ as an unknown quantity in the tortoise coordinate and demand that the Klein–Gordon equation can be reduced to the standard form as Eq. (17) near the horizon; then the parameter κ is determined. Equation (17) indicates that the parameter κ appears in the spectrum and is proportional to the radiation temperature; then the temperature of black hole is obtained.

From Eq. (12) we know that r_H is a function of θ on the horizon. So the tortoise coordinate transformation can be written as (Zhao and Dai, 1992)

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa(\theta_0)}\ln[r - r_H(\theta)], \\ \theta_* &= \theta - \theta_0, \end{aligned} \tag{18}$$

where θ_0 is an arbitrarily fixed parameter and does not vary in tortoise transformation. From Eq. (18) we can get

$$\begin{aligned} \frac{\partial}{\partial r} &= \left[1 + \frac{1}{2\kappa(r - r_H)}\right]\frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial\theta} &= \frac{\partial}{\partial\theta_*} - \frac{r'_H}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}; \end{aligned} \tag{19}$$

$$\frac{\partial^2}{\partial r^2} = \left[1 + \frac{1}{2\kappa(r-r_H)}\right]^2 \frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial\theta\partial r} = \left[1 + \frac{1}{2\kappa(r-r_H)}\right]\frac{\partial^2}{\partial\theta_*\partial r_*} - \frac{r'_H}{2\kappa(r-r_H)}\left[1 + \frac{1}{2\kappa(r-r_H)}\right]\frac{\partial^2}{\partial r_*^2} + \frac{r'_H}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial\theta^2} = \frac{r'^2_H}{[2\kappa(r-r_H)]^2}\frac{\partial^2}{\partial r_*^2} + \frac{\partial^2}{\partial\theta_*^2} - \frac{2r'_H}{2\kappa(r-r_H)}\frac{\partial^2}{\partial r_*\partial\theta_*} - \frac{r'^2_H + r''_H(r-r_H)}{2\kappa(r-r_H)^2}\frac{\partial}{\partial r_*}. \tag{20}$$

Substituting Eqs. (6), (7), (19), and (20) into the Klein–Gordon equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) - \mu^2\Phi = 0, \tag{21}$$

we can get the coefficient of $\frac{\partial^2\Phi}{\partial r_*^2}$ term

$$\frac{\{(1 - 2ar\cos\theta - \frac{2M}{r})[2\kappa(r-r_H)+1] - (2a\sin\theta)r'_H\}r^2[2\kappa(r-r_H)+1] + r'^2_H}{2\kappa(r-r_H)[2\kappa(r-r_H)+1]r^2} \tag{22}$$

If we demand that the Klein–Gordon equation reduces to the standard form as Eq. (17) near the horizon, the limit of Eq. (22) must be 1 when $r \to r_H(\theta_0)$ and $\theta \to \theta_0$. From this calculation of limit, we can deduce the expression of κ as

$$\kappa = \frac{1}{2r_H}\frac{\frac{M}{r_H^2} - a\cos\theta - \frac{r'^2_H}{r_H^3}}{\frac{M}{r_H^2} + a\cos\theta + \frac{r'^2_H}{2r_H^3}}. \tag{23}$$

Using Damour–Ruffini's approach (Damour and Ruffini, 1976), we can get the temperature of black hole as

$$T = \frac{\kappa}{2\pi} \quad \text{or} \quad \beta = \frac{2\pi}{\kappa}. \tag{24}$$

We can see that the temperature of the black hole is a function of θ.

For Rindler space–time, $M = 0$, temperature is a constant:

$$\kappa = a \quad \text{and} \quad \beta = \frac{2\pi}{a}. \tag{25}$$

4. THIN FILM BRICK-WALL MODEL

In the brick-wall model put forward by ’t Hooft (1985), the black hole entropy is identified with the entropy of the thermal gas of quantum field excitations outside the event horizon. This needs thermal equilibrium between the external fields and black hole. However, this qualification is not satisfied in the case of accelerating black hole, because the temperature on the horizon is not uniform as discussed in Section 3.

Recently, a new model, the thin film brick-wall model, is developed from the original brick-wall model (Liu and Zhao, 2001). This model considers that the entropy of a black hole should be only related to its horizon, because the event horizon is the characteristic surface of the black hole. Because of this opinion and the fact that the density of quantum states near the horizon is divergent, it is natural to take only the quantum field in a thin film near the event horizon into account. If we adopt the thin film brick-wall model, we find that although the global thermal equilibrium is not satisfied, local thermal equilibrium always exists. So the difficulty of the original brick-wall model is overcome.

We will calculate the entropy of the Rindler horizon relative to a uniformly accelerating observer in the next part of this section as an example of the thin film brick-wall model.

As we obtained in Section 2, the metric of the Rindler space–time for a uniformly accelerating observer is

$$\begin{aligned} ds^2 = &-(1 - 2ar\cos\theta - r^2a^2\sin^2\theta)\,dv^2 + 2\,dv\,dr \\ &- 2r^2a\sin\theta\,dv\,d\theta + r^2\,d\theta^2 + r^2\sin^2\theta\,d\varphi^2. \end{aligned} \tag{26}$$

The equation of the Rindler horizon is a paraboloid of revolution

$$r_H = \frac{1}{a(1+\cos\theta)}. \tag{27}$$

The solution of the Klein–Gordon equation has the following form (Lee and Kim, 1996; Ho *et al.*, 1997):

$$\Phi = e^{-i[Ev - m\varphi - S(r,\theta)]}. \tag{28}$$

With the WKB approximation, we have

$$g^{11}k_r^2 + 2(g^{12}k_\theta - Eg^{01})k_r + g^{22}k_\theta^2 + m^2g^{33} + \mu^2 = 0, \tag{29}$$

where

$$k_r = \frac{\partial S}{\partial r} \quad \text{and} \quad k_\theta = \frac{\partial S}{\partial \theta} \tag{30}$$

are radial wave number and angular wave number respectively. From Eq. (29) we can obtain the relationship between k_r and k_θ as

$$k_r^{\pm} = \frac{Eg^{01} - g^{12}k_\theta}{g^{11}} \pm \frac{\sqrt{(g^{12}k_\theta - Eg^{01})^2 - g^{11}(g^{22}k_\theta^2 + m^2 g^{33} + \mu^2)}}{g^{11}}, \quad (31)$$

According to quantum statistics theory, the free energy is expressed by (Ho *et al.*, 1997; Lee and Kim, 1996; Li and Zhao, 2000; Liu and Zhao, 2001; 't Hooft, 1985)

$$\begin{aligned} F &= -\frac{1}{4\pi^3}\int dm \int_0^{+\infty} dE \frac{1}{e^{\beta E}-1}\int d\theta\, d\varphi \int dk_\theta \left(\int_{r_H+\epsilon}^{r_H+\epsilon+\delta} k_r^+ dr \right. \\ &\quad \left. + \int_{r_H+\epsilon+\delta}^{r_H+\epsilon} k_r^- dr \right) \\ &= -\frac{1}{2\pi^3}\int dm \int_0^{+\infty} dE \frac{1}{e^{\beta E}-1}\int d\theta\, d\varphi \int dk_\theta \int_{r_H+\epsilon}^{r_H+\epsilon+\delta} \hat{k}_r\, dr, \end{aligned} \quad (32)$$

where

$$\hat{k}_r = \frac{\sqrt{(g^{12}k_\theta - Eg^{01})^2 - g^{11}(g^{22}k_\theta^2 + m^2 g^{33} + \mu^2)}}{g^{11}}. \quad (33)$$

Here we use the thin film brick-wall model (Liu and Zhao, 2001). In Eq. (32), ϵ is the cuttoff distance and δ is the thickness of the thin film.

First we study the integration with respect to k_θ and m. The expression in the radical sign of Eq. (33) should satisfy

$$(g^{12}k_\theta - Eg^{01})^2 - g^{11}(g^{22}k_\theta^2 + m^2 g^{33} + \mu^2) \geq 0. \quad (34)$$

This gives the integration limit of k_θ and m. We use little mass approximation in the process of integration, and the result is

$$F = -\frac{1}{6\pi^2}\int_0^{+\infty} dE \frac{E^3}{e^{\beta E}-1}\int d\theta\, d\varphi \int_{r_H+\epsilon}^{r_H+\epsilon+\delta} (g_{00})^{-2} r^2 \sin\theta\, dr. \quad (35)$$

After the integration with respect to E, we get the expression of the free energy as

$$F = -\frac{\pi^2}{90\beta^4}\int d\theta\, d\varphi \int_{r_H+\epsilon}^{r_H+\epsilon+\delta} (g_{00})^{-2} r^2 \sin\theta\, dr. \quad (36)$$

From

$$S = \beta^2 \frac{\partial F}{\partial \beta}\bigg|_{\beta=\beta_H}, \quad (37)$$

we obtain the expression of entropy as

$$S = \frac{4\pi^2}{90\beta_H^3} \int d\theta\, d\varphi \int_{r_H+\epsilon}^{r_H+\epsilon+\delta} (g_{00})^{-2} r^2 \sin\theta\, dr. \tag{38}$$

Considering the thin film brick-wall model, the integration on r can be processed as

$$\begin{aligned} S &\approx \frac{4\pi^2}{90\beta_H^3} \int r_H^2 \sin\theta\, d\theta\, d\varphi \int_{r_H+\epsilon}^{r_H+\epsilon+\delta} (1 - 2ar\cos\theta - r^2a^2\sin^2\theta)^{-2}\, dr \\ &= \frac{4\pi^2}{90\beta_H^3} \int r_H^2 \sin\theta\, d\theta\, d\varphi \int_{r_H+\epsilon}^{r_H+\epsilon+\delta} \frac{1}{a^4\sin^4\theta[r - \frac{1}{a(1+\cos\theta)}]^2[r + \frac{1}{a(1-\cos\theta)}]^2} dr \\ &\approx \frac{4\pi^2}{90\beta_H^3} \frac{1}{a^2} \frac{\delta}{\epsilon(\epsilon+\delta)} \frac{1}{4} \int r_H^2 \sin\theta\, d\theta\, d\varphi. \end{aligned} \tag{39}$$

Substituting Eq. (25) into Eq. (39), we get

$$S = \frac{1}{90\beta_H} \frac{\delta}{\epsilon(\epsilon+\delta)} \frac{1}{4} \int r_H^2 \sin\theta\, d\theta\, d\varphi. \tag{40}$$

Selecting appropriate ϵ and δ to satisfy

$$\frac{\delta}{\epsilon(\epsilon+\delta)} = 90\beta_H, \tag{41}$$

we have

$$S = \frac{1}{4} \int r_H^2 \sin\theta\, d\theta\, d\varphi = \frac{1}{4} \int dA, \tag{42}$$

where

$$dA = r_H^2 \sin\theta\, d\theta\, d\varphi. \tag{43}$$

This result indicates that the surface density of entropy on horizon is $\frac{1}{4}$. In next section, we will study the entropy of the uniformly acceleration black hole. The temperature on the horizon is not uniform. We will calculate the entropy density at every point of the horizon at first, and then we obtain the total entropy through integration.

5. ENTROPY OF UNIFORMLY ACCELERATING BLACK HOLE

In Section 2 we have obtained the metric of the uniformly accelerating black hole

$$\begin{aligned} ds^2 = &-(1 - 2ar\cos\theta - r^2a^2\sin^2\theta - 2Mr^{-1})\, dv^2 + 2dv\, dr \\ &- 2r^2a\sin\theta\, dv\, d\theta + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2 \end{aligned} \tag{44}$$

and the horizon equation

$$1 - 2ar_H \cos\theta - \frac{2M}{r_H} - (2a\sin\theta)r'_H + \frac{r'^2_H}{r^2_H} = 0. \tag{45}$$

We can see that the infinite redshift surface does not coincide with the event horizon. We expect that there exists a frame in which these two surfaces are identical.

Introduce a coordinate transformation

$$R = r - r_H(\theta), \qquad dR = dr - r'_H d\theta. \tag{46}$$

The metric becomes

$$\begin{aligned} ds^2 = &-(1 - 2ar\cos\theta - r^2a^2\sin^2\theta - 2Mr^{-1})\,dv^2 + 2\,dv\,dR \\ &- 2(r^2a\sin\theta - r'_H)\,dv\,d\theta + r^2\,d\theta^2 + r^2\sin^2\theta\,d\varphi^2. \end{aligned} \tag{47}$$

The form of metric can be changed to

$$\begin{aligned} ds^2 &= d_{00}\,dv^2 + 2\,dv\,dR + 2g_{02}\,dv\,d\theta + g_{22}\,d\theta^2 + g_{33}\,d\varphi^2 \\ &= \left(g_{00} - \frac{g^2_{02}}{g_{22}}\right)dv^2 + 2\,dv\,dR + \frac{g^2_{02}}{g_{22}}\left(\frac{g_{22}}{g_{02}}\,d\theta + dv\right)^2 + g_{33}\,d\varphi^2, \end{aligned} \tag{48}$$

where

$$g_{00} - \frac{g^2_{02}}{g_{22}} = -\left[1 - 2ar\cos\theta - \frac{2M}{r} - (2a\sin\theta)r'_H + \frac{r'^2_H}{r^2}\right]. \tag{49}$$

We introduce another coordinate transformation

$$d\Theta = \frac{g_{22}}{g_{02}}d\theta + dv. \tag{50}$$

The metric can be formally written as

$$ds^2 = \hat{g}_{00}\,dv^2 + 2\,dv\,dR + \hat{g}_{22}\,d\Theta^2 + \hat{g}_{33}\,d\varphi^2, \tag{51}$$

where

$$\hat{g}_{00} = g_{00} - \frac{g^2_{02}}{g_{22}}, \quad \hat{g}_{22} = \frac{g^2_{02}}{g_{22}}, \quad \hat{g}_{33} = g_{33}. \tag{52}$$

The following calculation about entropy is based on this metric. $\hat{g}_{00} = 0$ is just the surface equation of horizon.

Let us substitute the determinant and the contravariant components of metric into the Klein–Gordon equation, which describes the scalar field with mass μ,

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) = \mu^2\Phi. \tag{53}$$

We suppose that the solution has the following form (Ho *et al.*, 1997; Lee and Kim, 1996)

$$\Phi = e^{-i(Ev - m\varphi)} G(R, \Theta), \tag{54}$$

where

$$G(R, \Theta) = e^{is(R,\Theta)}. \tag{55}$$

With the WKB approximation, we have

$$\hat{g}^{11} k_R^2 - 2E k_R + \hat{g}^{22} k_\Theta^2 + m^2 \hat{g}^{33} + \mu^2 = 0, \tag{56}$$

where

$$k_R = \frac{\partial S}{\partial R}, \quad k_\Theta = \frac{\partial S}{\partial \Theta}. \tag{57}$$

From Eq. (56), we can obtain the relationship between k_R and k_Θ as

$$k_R^\pm = \frac{E}{\hat{g}_{11}} \pm \frac{\sqrt{E^2 - \hat{g}^{11}(\hat{g}^{22} k_\Theta^2 + m^2 \hat{g}^{33} + \mu^2)}}{\hat{g}^{11}}. \tag{58}$$

Free energy of the system is given by

$$F = -\int_0^{+\infty} dE \frac{\Gamma(E)}{e^{\beta E} - 1}, \tag{59}$$

where $\Gamma(E)$ is the total number of modes whose energy is not greater than E. Using the semiclassical quantization condition and the thin film brick-wall model, we have (Ho *et al.*, 1997; Lee and Kim, 1996; Li and Zhao, 2000; Liu and Zhao, 2001; 't Hooft, 1985)

$$\begin{aligned}\Gamma(E) &= \frac{1}{4\pi^3} \int dm \int d\Theta\, d\varphi \int dk_\Theta \left(\int_\epsilon^{\epsilon+\delta} k_R^+ dR + \int_{\epsilon+\delta}^{\epsilon} k_R^- \, dR \right) \\ &= \frac{1}{2\pi^3} \int dm \int d\Theta\, d\varphi \int dk_\Theta \int_\epsilon^{\epsilon+\delta} \hat{k}_R \, dR,\end{aligned} \tag{60}$$

where

$$\hat{k}_R = \frac{\sqrt{E^2 - \hat{g}^{11}(\hat{g}^{22} k_\Theta^2 + m^2 \hat{g}^{33} + \mu^2)}}{\hat{g}^{11}}. \tag{61}$$

The surface density of free energy on horizon can be expressed by

$$\sigma_F = -\int_0^{+\infty} dE \frac{\sigma_\Gamma}{e^{\beta E} - 1}. \tag{62}$$

σ_F and σ_Γ are defined as

$$F = \int \sigma_F \, dA \quad \text{and} \quad \Gamma = \int \sigma_\Gamma \, dA, \tag{63}$$

where

$$\begin{aligned} dA &= \sqrt{\hat{g}_{22}\hat{g}_{33}} \, d\Theta \, d\varphi \\ &= \left(r_H^2 a \sin\theta - r_H'\right) \sin\theta \, d\Theta \, d\varphi. \end{aligned} \tag{64}$$

Now, let us study the integration on k_Θ and m. We use little mass approximation in the process of integration, and the result is

$$\begin{aligned} \Gamma(E) &= \frac{E^3}{6\pi^2} \int d\Theta \, d\varphi \int_\epsilon^{\epsilon+\delta} (\hat{g}^{11})^{-2} (\hat{g}^{22}\hat{g}^{33})^{-\frac{1}{2}} \, dR \\ &= \frac{E^3}{6\pi^2} \int d\Theta \, d\varphi \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} (\hat{g}_{22}\hat{g}_{33})^{-\frac{1}{2}} \, dR \\ &\approx \frac{E^3}{6\pi^2} \int dA \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} \, dR \\ &= \int dA \, \frac{E^3}{6\pi^2} \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} \, dR. \end{aligned} \tag{65}$$

So the surface density of $\Gamma(E)$ is

$$\sigma_\Gamma = \frac{E^3}{6\pi^2} \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} \, dR. \tag{66}$$

The surface density of free energy is given by

$$\begin{aligned} \sigma_F &= -\frac{1}{6\pi^2} \int_0^{+\infty} dE \, \frac{E^3}{e^{\beta E} - 1} \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} \, dR \\ &= -\frac{\pi^2}{90\beta^4} \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} \, dR. \end{aligned} \tag{67}$$

From

$$S = \beta^2 \frac{\partial F}{\partial \beta}\bigg|_{\beta=\beta_H}, \tag{68}$$

we can obtain the surface density of entropy as

$$\sigma_S = \frac{4\pi^2}{90\beta_H^3} \int_\epsilon^{\epsilon+\delta} (\hat{g}_{00})^{-2} \, dR. \tag{69}$$

Because $\hat{g}_{00} = 0$ is the surface equation of horizon, $\hat{g}_{00}$ can be expressed by

$$\hat{g}_{00} = f(r, \theta)(r - r_H). \tag{70}$$

Substituting Eq. (70) into Eq. (69), we complete the integration on R as

$$\begin{aligned}\sigma_S &= \frac{4\pi^2}{90\beta_H^3}\int_\epsilon^{\epsilon+\delta}\frac{1}{f^2(r-r_H)^2}\,dR\\ &\approx \frac{4\pi^2}{90\beta_H^3 f_H^2}\int_\epsilon^{\epsilon+\delta}\frac{1}{R^2}\,dR\\ &= \frac{4\pi^2}{90\beta_H^3 f_H^2}\frac{\delta}{\epsilon(\epsilon+\delta)}.\end{aligned} \tag{71}$$

In the space–time which has the form of Eq. (51), the surface gravity of horizon is

$$\kappa = -\frac{1}{2}\lim_{R\to 0}\left(\frac{\partial \hat{g}_{00}}{\partial R}\right) = -\frac{f_H}{2}. \tag{72}$$

κ is just the value given by Eq. (23). Substituting Eq. (72) into Eq. (71), we get

$$\begin{aligned}\sigma_S &= \frac{4\pi^2}{90\beta_H^3}\frac{1}{(2\kappa)^2}\frac{\delta}{\epsilon(\epsilon+\delta)}\\ &= \frac{4\pi^2}{90\beta_H^3}\frac{1}{\kappa^2}\frac{\delta}{\epsilon(\epsilon+\delta)}\frac{1}{4}.\end{aligned} \tag{73}$$

Substituting $\beta_H = \frac{2\pi}{\kappa}$ into Eq. (73), we get

$$\sigma_S = \frac{1}{90\beta_H}\frac{\delta}{\epsilon(\epsilon+\delta)}\frac{1}{4}. \tag{74}$$

Selecting appropriate cutoff distance ϵ and thickness of thin film δ to satisfy

$$\frac{\delta}{\epsilon(\epsilon+\delta)} = 90\beta_H, \tag{75}$$

we can obtain the surface density of entropy as

$$\sigma_S = \frac{1}{4}. \tag{76}$$

The total entropy is

$$S = \int \sigma_S\,dA = \frac{1}{4}A_H. \tag{77}$$

6. CONCLUSION

By using the thin film brick-wall model, we have studied the entropy of the uniformly accelerating black hole. We get our metric by simplifying Kinnersley's metric. The black hole is uniformly accelerated and the mass does not vary with time. Because the black hole is accelerated, the horizon is axisymmetric. We use a new method to calculate the temperature of black hole at every point on the horizon, and find that the temperature of black hole is the function of θ. That is to say, the different points of horizon surface may have different temperatures. To overcome the difficulty encountered in standard brick-wall model, we adopt the thin film brick-wall model in which only the local thermal equilibrium is needed. We calculate the entropy density at every point of the horizon at first, and then we obtain the total entropy through integration. The calculation indicates that the conclusion that black hole entropy is proportional to its area can be applied to horizon not only globally, but also locally.

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